**Analysis of Two-factor Designs**

A two-factor analysis of [variance](http://davidmlane.com/hyperstat/A128518.html) consists of three significance tests: a test of each of the two [main effects](http://davidmlane.com/hyperstat/A131633.html) and a test of the [interaction](http://davidmlane.com/hyperstat/A130844.html) of the variables. An analysis of variance [summary table](http://davidmlane.com/hyperstat/B91476.html) is a convenient way to display the results of the significance tests. A summary table for the hypothetical experiment described in the section on [factorial designs](http://davidmlane.com/hyperstat/A134930.html) and a graph of the means for the experiment are shown below.

Sum of Mean

SOURCE df Squares Square F p

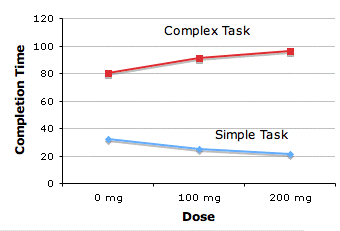
T 1 47125.3333 47125.3333 384.174 0.000

D 2 42.6667 21.3333 0.174 0.841

TD 2 1418.6667 709.3333 5.783 0.006

ERROR 42 5152.0000 122.6667

TOTAL 47 53738.6667

  
  
**Sources of Variation**  
The summary table shows four sources of variation: (1) Task, (2) Drug dosage, (3) the Task x Drug dosage interaction, and (4) Error.

**Degrees of Freedom**  
The [degrees of freedom](http://davidmlane.com/hyperstat/A42408.html) total is always equal to the total number of numbers in the analysis minus one. The experiment on task and drug dosage had eight subjects in each of the six groups resulting in a total of 48 subjects. Therefore, df total = 48 - 1 = 47.  
  
The degrees of freedom for the main effect of a factor is always equal to the number of [levels](http://davidmlane.com/hyperstat/A36135.html) of the factor minus one. Therefore, df task = 2 - 1 = 1 since there were two levels of task (simple and complex). Similarly, df dosage = 3 - 1 = 2 since there were three levels of drug dosage (0 mg, 100 mg, and 200 mg).   
  
The degrees of freedom for an interaction is equal to the product of the degrees of freedom of the variables in the interaction. Thus, the degrees of freedom for the Task x Dosage interaction is the product of the degrees of freedom for task (1) and the degrees of freedom for dosage (2). Therefore, df Task x Dosage = 1 x 2 = 2.  
  
The degrees of freedom error is equal to the degrees of freedom total minus the degrees of freedom for all the effects. Therefore, df error = 47 - 1 - 2 - 2 = 42.

**Sums of Squares**  
Computational formulas for the sums of squares will not be given since it is assumed that complex analyses will not be done by hand.  
  
**Mean Squares**  
As in the case of a one-factor [design,](http://davidmlane.com/hyperstat/B91801.html) each mean square is equal to the sum of squares divided by the degrees of freedom. For instance, Mean square dosage = 42.6667/2 = 21.333 where the sum of squares dosage is 42.6667 and the degrees of freedom dosage is 2.  
  
**F Ratios**  
The F ratio for an effect is computed by dividing the mean square for the effect by the mean square error. For example, the F ratio for the Task x Dosage interaction is computed by dividing the mean square for the interaction ( 709.3333) by the mean square error (122.6667). The resulting F ratio is: F = 709.3333/122.6667 = 5.783

**Probability Values**  
To compute a [probability value](http://davidmlane.com/hyperstat/A131927.html) for an F ratio, you must know the degrees of freedom for the F ratio. The degrees of freedom numerator is equal to the degrees of freedom for the effect. The degrees of freedom denominator is equal to the degrees of freedom error. Therefore, the degrees of freedom for the F ratio for the main effect of task are 1 and 42, the degrees of freedom for the F ratio for the main effect of drug dosage are 2 and 42, and the degrees of freedom for the F for the Task x Dosage interaction are 2 and 42.  
  
An [F distribution calculator](http://davidmlane.com/hyperstat/F_table.html) can be used to find the probability values. For the interaction, the probability value associated with an F of 5.783 with 2 and 42 df is 0.006.   
  
**Drawing Conclusions**  
When a main effect is significant, the [null hypothesis](http://davidmlane.com/hyperstat/A29337.html) that there is no main effect in the [population](http://davidmlane.com/hyperstat/A10626.html) can be rejected. In this example, the effect of task was significant. Therefore it can be concluded that, in the population, the mean time to complete the complex task is greater than the mean time to complete the simple task (hardly surprising). The effect of dosage was not significant. Therefore, there is no convincing evidence that the mean time to complete a task (in the population) is different for the three dosage levels

The significant Task x Dosage interaction indicates that the effect of dosage (in the population) differs depending on the level of task. Specifically, increasing the dosage slows down performance on the complex task and speeds up performance on the simple task. The effect of increasing the dosage therefore depends on whether the task is complex of simple.  
  
There will always be some interaction in the sample data. The significance test of the interaction lets you know whether you can infer that there is an interaction in the population.